Marburg University

Department of Computer Science
Prof. Dr. Sebastian Wild

Date: 2026-06-26
Version: 2026-06-25 23:31

Exercise Sheet 10 for Advanced Data Structures (Summer 2026)

Hand In: *Until 2026-06-03 18:00, on ILIAS.*

Problem 1

30 points

You are given a fixed rooted tree T with vertices $1, \dots, n$, and must create a data structure which maintains a set S of vertices dynamically. Your data structure must be able to quickly answer the following type of query: "Given a vertex x, how many vertices y in the set are on the path from the root to x."

The worst-case running time of operating on your data structure must be at most $\log \log n$ times larger than the optimal amortised running time.

Problem 2

30 points

a) Give a (very simple!) data structure which maintains a set of elements, where the operations are (i) inserting an element, (ii) undoing an insertion, (iii) retrieving the minimum element. All operations should take $O(1)$ time.

b) Explain how to simulate a *double ended queue*, i.e. a data structure that maintains a sequence of elements and allows inserting and deleting from the left and right side of the sequence, using 3 stacks.

Hint: One stack is used only for rebalancing!

c) Using the last 2 sections, give a data structure that maintains a double ended queue of elements, and additionally can output the minimum element in the double ended queue. All operations should take $O(1)$ amortised time.

Hint: A stack is a data structure that allows push (insert) and pop (undo insert)!

d) Generalise this to the following, more complicated query. Suppose you are given a data structure which, as a black-box, maintains a set of S *linear functions*, i.e. functions of the form $x \mapsto ax + b$, allowing insertion and undoing insertions in $T(n)$ time. Furthermore, this black-box data structure can also answer the following type of query: given x, find $\min_{f \in S} f(x)$ — in other words, of the set of maintained linear function, output the minimal value of a linear function when evaluated at x. This operation is also in $T(n)$ time.

Now, give a data structure which maintains a double ended queue of linear functions, allowing inserting and deleting from the left and right end of the queue, and furthermore answering queries of the following type: "Given x, and if the linear functions in the double ended queue are $f_1, \ldots, f_n$, output $\min_i f_i(x)$." All operations should take amortised $O(T(n))$ time.